

## DIRECTIVE MOTION OF A MULTIPLE-CONSTAINED HOLOMOMIC SYSTEM

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**Abstract:** Within this paper, we propose an analytical study for new conveying system. This system is flexible of all directions for input and output distribution. Our active conveying system consists of multi-arrays of rotating wheels or cells. Each cell is a ball driven by two D.O.F. mechanisms. The orientation and speed are controlled using a concept of distributed network controllers. These arrays can be operated to manipulate parcels towards desired directions. Each cell has its own controller that communicates with neighboring cells to simultaneously follow desired trajectory. Motion characteristics look like flocking patterns of birds flying using simple rules between independent agents reaching their directions. Orientation and direction changing of parcels directly depend on summing of force and moment that cells act to it. We derive model of Kinematics and Dynamics with the presence of multiple constraints. Both models are used for parcel manipulation and control.

### Nomenclatures:

- $r_p$  : Parcel position.
- $\dot{r}_p$  : Parcel velocity.
- $r_i$  : Cell coordinate.
- $\bar{r}_i$  : Cell array coordinate.
- $V_i$  : Spinning velocity of cell  $i$ .
- $\omega_p$  : Angular velocity of parcel.
- $\mu_k$  : Kinetics friction coefficient.
- $\tau_i$  : Applied torque of wheel of cell  $i$ .
- $R$  : Wheel radius.
- $N_i$  : Normal forces at cell  $i$ .
- $\hat{f}_i$  : Driving Force.
- $\hat{v}_{rel_i}$  : Unit vector of relative velocity between cell and parcel at contact point
- $\hat{v}_i$  : Direction of spinning velocity
- $I_p$  : Mass moment of inertia of parcel
- $m$  : Mass of parcel
- $W$  : Weight of parcel
- $K_L$  : Linear spring
- $K_{\theta_x}$  : Angular spring in X-axis
- $K_{\theta_y}$  : Angular spring in Y-axis
- $d_{xi}$  : Distance from center of cells group to cell  $i$  in X-axis
- $d_{yi}$  : Distance from center of cells group to cell  $i$  in Y-axis
- $\phi$  : Potential function
- $x_d, y_d$  : Destination position

$B_p$  : Boundary function

### 1. INTRODUCTION

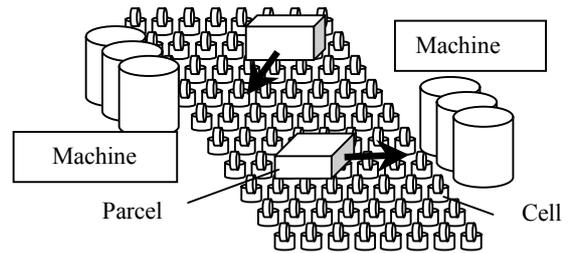


Fig. 1 Concept of the new conveyor system

Global competition urges industry to better use technology in order to increase quality and reduce cost. Many plants are now investigating a possibility of using a Flexible Manufacturing System (FMS). One feature of this system is an ability to manipulate parts with arbitrary translational and rotational directions as shown in Fig 1. Prior work that related to this research are as follow: S. Konishi [1] propose a small parts conveying system using fluidic micro actuators without feedback sensor. A positioning method allows every actuator to exert forces to desirable directions. K. Bohringer [2] presented a method for manipulating very small parts by electrostatic forces supplied by actuator arrays. Orientation and position of parts are controlled by proper electrostatic fields. W. Messner [3] experimented in manipulating heavy parcels in plane by using multi-wheel actuators. Elliptic, partial differential equations were applied to control actuator's velocity.

The work [1] and [2] were about manipulating low weight and small parts in electronics industry. Although Messner's work can deal with manipulation of larger parts, here is no part explaining behavior of contact points between supporting wheels and parcels. Applied forces of actuators are treated as kinetic friction forces at all time. Directions of forces were determined by vectors of wheel velocity. In reality, the directions of applied forces do not depend on directions of wheel velocity. They depend on relative velocity at contact points.

Our study is an extension of Messner's work. We focus on the effect of such relative velocity generated at contact points. We have designed and built a new configuration for an active conveying system which will be described in the next section.

### 2. DESIGN CONFIGURATION

As depicted in Fig 2, a cell consists of a wheel and a steering mechanism. It generates velocity in any direction.



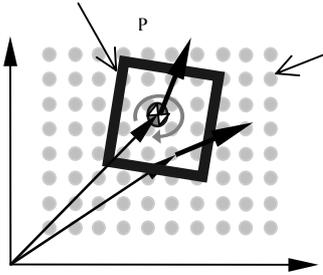
(a) One cell unit designed and fabricated at FIBO



(b) Array of cells at FIBO

Fig 2 A new conveying system.

We will perform kinematics and dynamics analysis on such a configuration as follow:



**2.1 Analysis of Kinematics.** Towards a goal of representing the designed velocity of parcel on the plane of arrays of rotating wheels so call “cell”, we now derive velocity equations for each cell. Understanding of these physical relations will enable us to maintain translational and rotational velocity of the parcel ( $V_p, \omega_p$ ) along of parcel along designed path.

Fig 3. Motion of a parcel in a multi actuator systems

Consider Fig 3, we assign a world coordinate having an origin at point (0,0). We further define the following parameter:

Parcel position ( $r_p$ ):

$$r_p = \begin{bmatrix} x_p \\ y_p \end{bmatrix}, \quad (1)$$

Parcel velocity ( $\dot{r}_p$ ):

$$\dot{r}_p = \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix},$$

Cell coordinate ( $r_i$ ):

$$r_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \quad (2)$$

Cell array coordinate ( $\bar{r}_i$ ):

$$\bar{r}_i = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \end{bmatrix}, \quad \text{and} \quad (3)$$

Spinning velocity of cell ( $V_i$ ):

$$V_i = \dot{r}_p + \omega_p \times (r_i - r_p) \quad (4)$$

Note that subscript p and i represent parcel and number of cells respectively the maximum number of cell is n. In matrix form, we rewrite equation (4) as:

$$[V_i]_{2 \times n} = \dot{r}_p [1]_{1 \times n} + \omega_p \left\{ (\bar{r}_i - r_p [1]_{1 \times n})^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}^T \quad (5)$$

For maintaining a kinematics value of parcel, most cells must develop a right velocity that might be inconstant along motion of parcel. We plot magnitude of  $V_i$  with respect to position x,y using equation (5),  $\dot{r}_p = 0.5$  m/s,  $\omega_p = 0$  rad/s, shown in Fig 4.

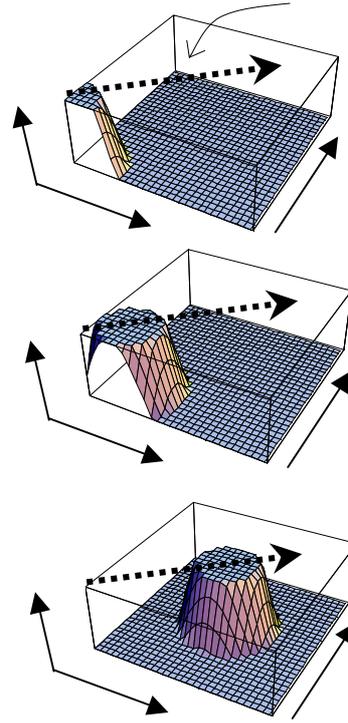


Fig 4. A Plot of velocity magnitude of parcel using eq.(5)

**2.2 Analysis of Dynamics.** In order to move parcels with desired translational and rotational velocities, we have to understand forces provided by cells to the parcel. Linear momentum and angular momentum changing of parcel are a result of forces and moments which cells apply to it. Because of a discontinuous supporting, a number of parcel and contact positions which relative to center of mass of parcel will vary at all time, the moment of each applied force is changed with position of parcel. At this reason, we must know behavior of contact force and a relationship between applied force (by cell) and a receiving force and a receiving moment that occur at parcel.

**2.2.1 Effect of Coulomb Friction.** With the presence of Coulomb friction, horizontal forces are generated by cells. The contact between cells and the parcel can be divided as stick rolling contact and slip rolling contact. In case of stick rolling contact, forces occur when relative velocity ( $v_{rel}$ ) equals to zero and the opposite situation ( $v_{rel} \neq 0$ ), force is limited to kinetic friction forces.

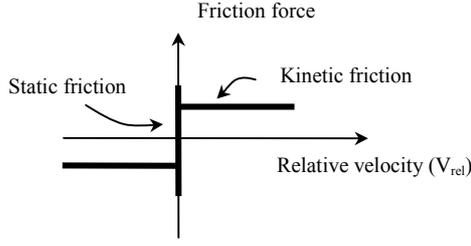


Fig 5. Friction and relative velocity relationship

To illustrate a friction force for a slipping case in multiple contact point in Fig 6, the force at slipping contact point equal kinetic friction force and its direction respect to relative velocity between parcel and cell.

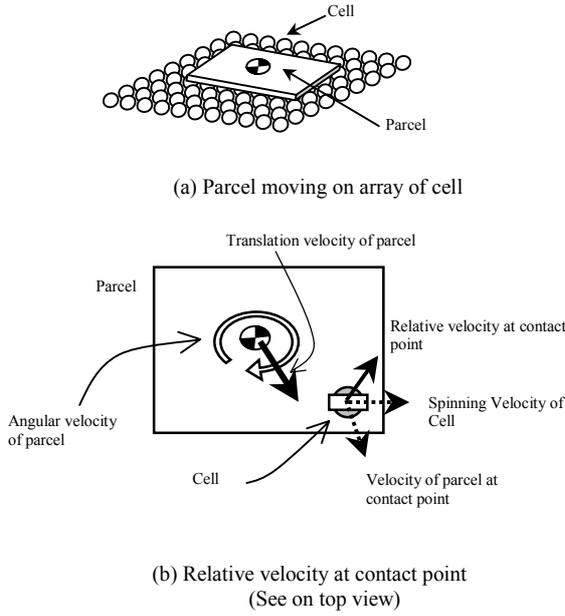


Fig. 6 Relative velocity diagram at Contact point

**2.2.2 Changes of Linear Momentum.** Parcel movement is a result of apply forces form an instant supporting cells. Linear momentum changing direct varies with a number of force and its magnitude that can be defined as:

$$m\ddot{r}_p = \sum_{i=1}^n F_i \quad (6)$$

when  $F_i$  is the apply force form cell  $i$  that consist of friction force and driving force form the cell  $i$ , defined as:

$$F_i = \mu_k N_i \hat{v}_{rel_i} + f_i \quad (7)$$

$$f_i = \text{Sgn}(\hat{v}_{rel_i}) \tau_i R \hat{v}_i \quad (8)$$

and

$$\text{Sgn}(\hat{v}_{rel_i}) = \begin{cases} 0 & \text{if } (\hat{v}_{rel_i}) > 0 \\ 1 & \text{if } (\hat{v}_{rel_i}) = 0 \end{cases}$$

where as

- $m$  : Mass of parcel.
- $\mu_k$  : Kinetics friction coefficient.
- $\tau_i$  : Apply torque of wheel of cell  $i$ .

$R$  : Wheel radius.

$N_i$  : Normal force at cell  $i$ .

$f_i$  : Driving force.

$\hat{v}_{rel_i}$  : Unit vector of relative velocity between cell and parcel at contact point and.

$\hat{v}_i$  : Direction of spinning velocity.

**2.2.3 Changes of Angular Momentum.** The relative position between cell and parcel is not constant when parcel has a linear velocity, the applied moment will be changed a magnitude although the applied force is constant. The relation between angular momentum of parcel and the applied force can express as:

$$I_p \dot{\omega}_p = \sum_{i=1}^n (r_i - r_p) \times F_i \quad (9)$$

when

$I_p$  : Mass moment of inertia of a parcel.

Motion simulation of parcel on array of cell using equation (6) and (9) with constant velocity 0.5 m/s (show by small arrow) by each parameter, mass = 5 kg, friction coefficient = 0.2, and parcel diameter = 11 cm is shown in Fig 7.

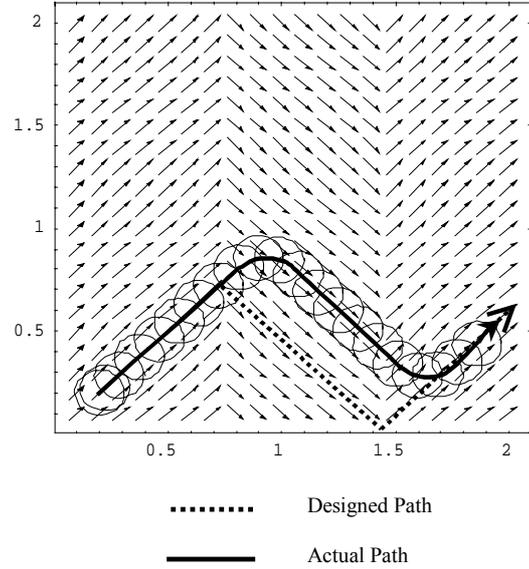


Fig 7 Motion of parcel with slip rolling force

Simulation result shows an error of path when contact forces were kinetic forces only. The direction of parcel does not simultaneous track to spinning direction of cells if we treat the apply forces as kinetics friction force only.

In fact, there are two modes in manipulating parcels namely stick and slip rolling contact. Note that applied forces in the first mode is greater than ones in the second mode. To guarantee stick rolling mode for all cell for all motion of parcel, the maximum applied forces must be smaller than sticking rolling forces that depend on friction coefficient and normal force ( $N_i$ ). Normal forces could be derived by the method of Tree Spring Model (TSM) depicted in Fig 8. Our assumptions are:

- Every support is treated as a spring.
- The parcel has rigid planar bottom face.
- Every contact between cell and parcel is maintained during inclination shown in Fig 7.

- The Tree Spring Model requires a linear spring ( $K_L$ ), an angular spring on x-axis ( $K_{\theta_x}$ ) and an angular spring on y-axis ( $K_{\theta_y}$ ).

- Spring constants are defined as:

$$K_{\theta_x} = \sum_{i=1}^n K_i d_{yi}^2,$$

$$K_{\theta_y} = \sum_{i=1}^n K_i d_{xi}^2,$$

$$K_L = \sum_{i=1}^n K_i,$$

$d_{xi}$  is distance from center of cells group ( $x_{ccg}$ ) to cell i in X-axis and

$d_{yi}$  is distance from center of cells group ( $y_{ccg}$ ) to cell i in Y-axis.

- $W$  is the weight of parcel.
- The angle of inclination of parcel with respect to X and Y-axis is very small.

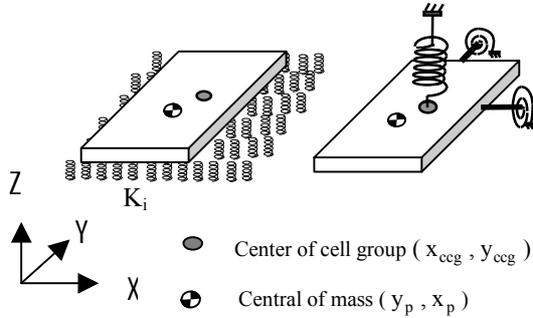


Fig 8 Tree Spring Model

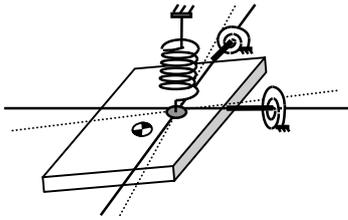


Fig 9 Inclination of a parcel after contacting with cells

Normal force ( $N_i$ ) can be obtained from eq (10),  $y_p$  and  $x_p$  are coordinates of center of mass of parcel and  $n$  is number of supporting cells, we have found

$$N_i = W \cdot (y_p / \sum_{i=1}^n d_{yi}^2 + x_p / \sum_{i=1}^n d_{xi}^2) + W / n. \quad (10)$$

To simplify equation (10), we define

$$A_i = W / \sum_{i=1}^n d_{xi}^2, B_i = W / \sum_{i=1}^n d_{yi}^2 \text{ and } C_i = W / n.$$

With a simple mathematical manipulation, we obtain

$$N_i = A_i(x_p) + B_i(y_p) + C_i \quad (11)$$

We rewrite equation (11) in a matrix involving all cells:

$$[N_i]_{n \times 1} = [A_i]_{n \times 1} x_p + [B_i]_{n \times 1} y_p + [C_i]_{n \times 1} \quad (12)$$

We use equation (12) to compute  $F_i$  and solve the motion of the parcel both sticking and slipping modes as results of multi-cell driven manipulation. In addition, we set the force resulting in the sticking mode as an upper bound of driving force from cells to the parcel. A corresponding controller, using such an upper bound as a limiting value to control position and orientations of the parcel to desired trajectories.

### 3.DISTRIBUTED CONTROL ALGORITHM

As clearly indicated in previous section our new active conveying system require multi actuator to provide driving forces. Each actuator has its own local controller, Intel N87C196MH. The controller is capable of communicating four neighboring cells through the RS232 serial port and multiplexers. Initial setpoints, given by a host PC, are simultaneously broadcasted to each cell by RS422. The broadcasted informations are parcel ID, initial position, destination, velocity and the width of active cell group. Once the ID is matched, the local controller will use received data to coordinate with neighborhoods, such that multi cells can simultaneously support the same parcel. Group of cells will calculate direction, velocity and path width by using the same functions. These functions can be expressed in three equations as follow:

**3.1 Uniform Velocity Equation.** The potential field, proposed by Oussama Khatip[10] and Jim-Oh Kim [11]., are used in several work related to obstacle avoidance. We further modified the potential field to suit the modeling and designs of our active conveying system. In this work, every parcel has its own destination and trajectory which sometime causes intersection and collision among parcels. We propose a velocity function base on potential function [10] as follow:

$$V_x = \left[ \frac{x_d - x^*}{\sqrt{(x_d - x^*)^2 + (y_d - y^*)^2}} \right] U - \frac{\partial \phi}{\partial x^*} \quad \text{and} \quad (13)$$

$$V_y = \left[ \frac{y_d - y^*}{\sqrt{(x_d - x^*)^2 + (y_d - y^*)^2}} \right] U - \frac{\partial \phi}{\partial y^*} \quad (14)$$

When

$U$  is designed velocity,

$V_x$  is velocity of cell in X-direction,

$V_y$  is velocity of cell in Y-direction,

$\phi$  is potential function,

$x_d, y_d$  is destination Position and

$x^*, y^*$  is Parcel position that observed from sensor of cell group.

Equation (13) and (14) will generate paths from current positions of parcel towards destination. Parcel position is detected by all four sensors surrounding each cell.

**3.2 Path boundary function.** Base on stream function, we propose a path boundary function as follow:

$$B_i = V_x \cdot y_i - V_y \cdot x_i, \quad (15)$$

$$B_p = V_x \cdot y_{p_i} - V_y \cdot x_{p_i}, \quad (16)$$

when

$x_i, y_i$  is position of cell i and

$x_{p_i}, y_{p_i}$  is position of parcel  $i$ .

Every cells will calculate its own boundary value ( $B_i$ ) by equation (14) and compare it with path bound ( $B_{Upper}$  and  $B_{Lower}$ ). The cell will active if its boundary value is in boundary range ( $B_{Lower} \leq B_i \leq B_{Upper}$ ). The upper and lower bound are defined as:

$$B_{Upper} = B_p + \Delta B \quad \text{and} \quad (17)$$

$$B_{Lower} = B_p - \Delta B \quad (18)$$

The path width depends on dimension of the parcel, safe distance and velocity of the parcel. The value of  $\Delta B$  can formulate by geometry relationships as equation (19)

$$\Delta B = r \sqrt{V_x^2 + V_y^2}, \quad (19)$$

when

$r$  is a tangential distance measured from the center of parcel to the upper or lower bound.

Note that the path width is  $2 \Delta B$ . The Boundary function has an orthogonal property with the potential function in equation (13) and (14). Path boundary will be generated by this function. Information on velocity and destination of each parcel will not flow out of the path boundary. Path boundary can be integrated with many useful behaviors by using potential fields for collision avoidance and etc.

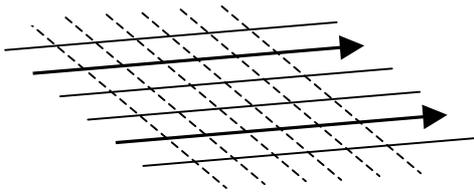
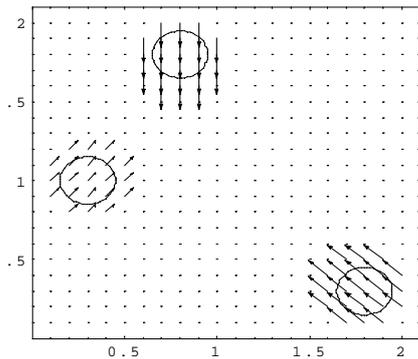
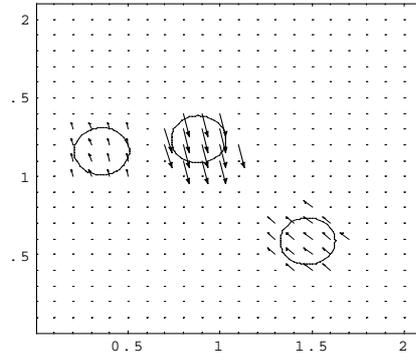


Fig 10 Show boundary of path under Boundary function

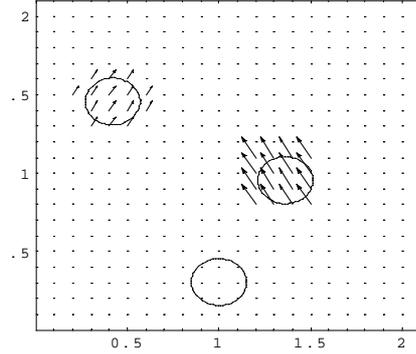
Using equation (13)-(18), we perform a simulation of three circular parcels on  $20 \times 20$  array of actuators is shown as follow, distance between cell is 10 cm, parcel diameter 30 cm, designed velocity ( $U$ ) is 0.5 m/s. The result is graphically in Fig 11.



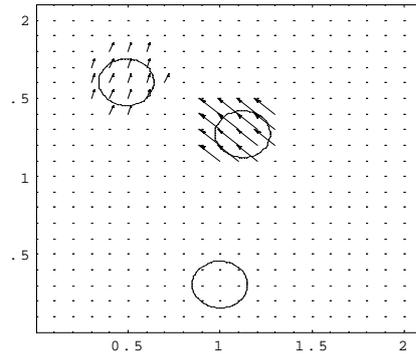
(a) Every parcel starts moving toward their goal. (time = 0 sec)



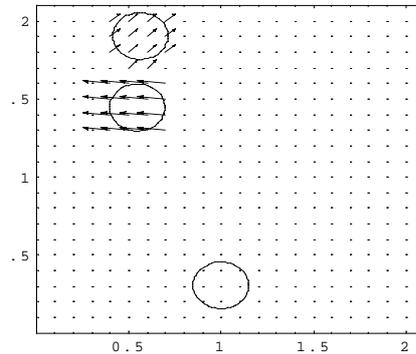
(b) The parcel 1 and 2 change their direction to avoid collision (time = 1 sec)



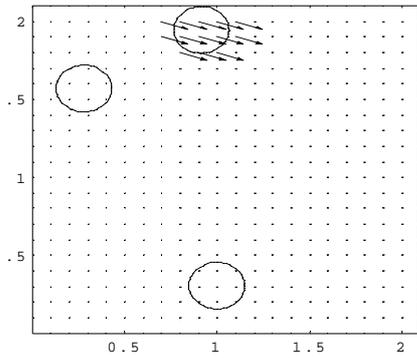
(c) The parcel 1 arrives at destination. (time = 2 sec)



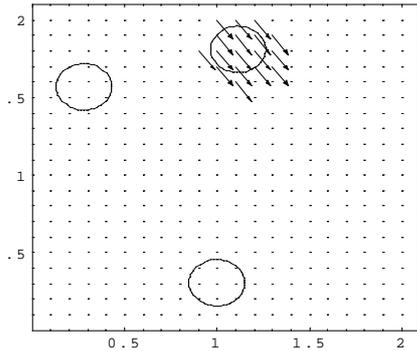
(d) The parcel 2,3 try to reach their goal. (time = 2.5 sec)



(e) Collision avoidance of two parcels (2,3). (time = 3.5 sec)



(f) The parcel 3 reach its destination .  
(time = 4 sec)



(g) The last parcel tries to reach its goal.  
(time = 5 sec)

Fig 11 Simulation of moving of multiple parcel.

#### 4.CONCLUSION AND FUTURE WORK

In this paper, we have proposed a method of manipulating parcels on multi-actuator array. Our analytical dynamics is the extension Messner's work. We also develop the method of group coordination of multiple processors and path boundary function. The effectiveness of these method and function are simulated.

We have finished designed and fabricated 16 cell array and will further implement our methods.

#### 5.ACKNOWLEDGEMENT

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Mr. Satadaechakul graduated from King Mongkut's Institute of Technology Thonburi(KMUTT) with a Bachelor Degree in Mechanical Engineering. Analysis of Combustion Process from Engine Pressure Cylinder was his interest. Currently he is a graduate student at KMUTT, performing research on distributed control. He is a researcher assistant at Center of Operation for Field Robotics Development (FIBO) at KMUTT, Thailand.

