A high performance haptic system should provide both stable and realistic interaction to users. In order to improve system stability, virtual couplings have been used in order to separate the haptic display from virtual environment simulations. A static virtual coupling can improve system stability but unfortunately degrades realistic performance of the system. This improved concept named adaptive virtual coupling is designed such that its parameters can be adjusted to maximize realistic performance while ensuring stability. The experimental result indicated that with adaptive virtual coupling users could feel more realistic interaction than the existing static virtual coupling method.

Keywords: Haptic System, Realistic Performance, Virtual Reality

2. Virtual Coupling

Consider a linear one-degree of freedom impedance display haptic system as shown in figure 1.

![Figure 1 Impedance Display Haptic System](image-url)

From figure 1, $F_h$, $v_h$, $F_e$ and $v_e$ represent human force, human velocity, virtual environment force and virtual environment velocity, respectively. The parameters with “*” are in discrete form. The haptic system can be modeled in form of Two-port representation as

$$
\begin{bmatrix}
F_h \\
-v_e
\end{bmatrix} =
\begin{bmatrix}
Z_d(z) & ZOH(z) \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
v_h \\
F_e
\end{bmatrix}
$$

In order to find the unconditionally stability criteria of a two-port network haptic system, the human operator and the virtual environment should be assumed to be passive operators. From Llewellyn’s stability criteria [6], typical haptic system would not be unconditional stable. However, it does not mean that the system will always unstable. Therefore, Colgate and his colleague [2] proposed a virtual coupling which is a synthetic coupling. This coupling, which will be described in the next subsection, is used as a complementary component in order to make the system satisfy unconditionally stability criteria.
A virtual coupling, a synthetic coupling which is shown in figure 2, is used for connecting between the haptic device and the virtual environment in order to improve the stability of haptic interface system.

Figure 2. Schematic Diagram of a Haptic System with a Virtual Coupling

This coupling could be modeled as any arbitrary structure. As shown in figure 2, a spring/damper model is used as our coupling. The impedance of the virtual coupling is

$$Z_c(z) = \left( b_c + \frac{k_c}{s} \right) s \rightarrow \frac{1}{Tz}$$  \hspace{1cm} (2)

The two-port representation of the haptic system with a virtual coupling can be written as

$$\begin{bmatrix} F_h \\ -v_e^* \end{bmatrix} = \begin{bmatrix} Z_d(z) & ZOH(z) \\ -1 & 1/Z_c(z) \end{bmatrix} \begin{bmatrix} v_h \\ F_e^* \end{bmatrix}$$  \hspace{1cm} (3)

Based on Llewellyn’s stability criteria [6], this haptic system with a virtual coupling would be an unconditionally stable system if

$$\text{Re}(Z_d(z)) \geq 0,$$  \hspace{1cm} (4)

$$\text{Re}(1/Z_c(z)) \geq 0,$$  \hspace{1cm} (5)

$$\text{Re}(1/Z_c(z)) \geq \frac{1 - \cos \angle \text{ZOH}(z)}{2 \text{Re}(Z_d(z))} |\text{ZOH}(z)|$$  \hspace{1cm} (6)

From equation (6), the virtual coupling could be designed to satisfy the unconditionally stability criteria.

3. Adaptive Virtual Coupling

Even though the use of the pre-calculated virtual coupling can ensure system stability when the human operator and the virtual environment are passive, but at the same time the realistic performance is affected. Thus, the compromising solution between stability and realistic performance of haptic system can be found by adapting the virtual coupling parameters. The adaptive virtual coupling has parameters that are adapted according to interaction frequency to satisfy both stability condition and realistic performance during operation.

The realistic performance boundaries, proposed by Lertpolpairoj and his colleagues [10], represent variation of human perceived impedance ($Z_h(z)$) while stably interacting with the reference environment through a haptic interface without virtual coupling.

These boundaries depend on impedance of an adjustable virtual environment. They could be delineated from an experimental data. The upper and lower values of this range are substituted into the human perceived impedance equation, equation (1), and the plot of realistic performance boundaries can be obtained as shown in figure 3.

Figure 3. Realistic Performance Boundaries of a Haptic Interaction

adapting virtual coupling is

$$Z_{ave}(z) = \left( b_{ave}(z) + \frac{k_{ave}(z)}{s} \right) s \rightarrow \frac{1}{Tz}$$  \hspace{1cm} (7)

Thus, the human perceived impedance of haptic system with an adaptive virtual coupling ($Z_{h\_ave}(z)$) can

$$Z_{h\_ave}(z) = Z_d(z) + ZOH(z)Z_{e\_ave}(z)$$  \hspace{1cm} (8)

where $Z_{e\_ave}(z) = \frac{Z_c(z)Z_{ave}(z)}{Z_c(z) + Z_{ave}(z)}$  \hspace{1cm} (9)

Suppose that we want to find an adaptive virtual coupling of a virtual environment. The first set of parameters that should be determined are the parameters of the realistic performance boundaries such as the damping and stiffness of the upper bound ($b_{e\_UCI}$ and $k_{e\_UCI}$) and the damping and stiffness of the lower bound ($b_{e\_LCI}$ and $k_{e\_LCI}$).

The desired human perceived impedance

$$Z_{h\_ave}(z) = Z_d(z) + ZOH(z)Z_{e\_ave}(z)$$  \hspace{1cm} (8)

where $Z_{e\_ave}(z) = \frac{Z_c(z)Z_{ave}(z)}{Z_c(z) + Z_{ave}(z)}$  \hspace{1cm} (9)

The desired human perceived impedance
\( Z_{h\_ds}(z) \), which represents these boundaries as shown in equation (10), can be formulated:

\[
Z_{h\_ds}(z) = Z_d(z) + ZOH(z)Z_{ds}(z) \tag{10}
\]

where the desired impedance is

\[
Z_{ds} = \left( \frac{b_{e\_UCI} + b_{e\_LCT}}{2} + \frac{k_{e\_UCI} + k_{e\_LCT}}{2s} \right) \left( \frac{T(z-1)}{2(z+1)} \right)^2
\]

The desired impedance is defined as the impedance, which its magnitude is laying in the middle between the upper bound and the lower bound. Thus, the parameters of \( Z_{ds} \) are the average parameters between the parameters of these two bounds. Reminding that the goal of the adaptive virtual coupling is to maximize the realistic performance while ensuring system stability. Hence, this goal can be mathematically formulated as:

To minimize:

\[
\left( Z_{h\_ds}(z) - Z_{h\_ave}(z) \right)^2 \tag{11}
\]

Subject to constraint:

\[
\text{Re}(Z_{h\_ave}(z)) \geq 0, \quad \forall \omega \in \mathbb{R} \tag{12}
\]

However, it is not easy to use \( b_{ave} \) and \( k_{ave} \) as the optimized variables because the impedance equations, i.e. human perceived impedance of haptic system with adaptive virtual coupling \( Z_{h\_ave}(z) \) and \( Z_{h\_ds}(z) \), are all complex functions. Moreover, our constraint, which is described in form of

\[
\text{Re}(Z_{h\_ave}(z)), \text{cannot be easily represented in } b_{ave} \text{ and } k_{ave}.
\]

The feasible region of the solution (feasible \( b_{ave} \) and \( k_{ave} \) that satisfy the objective function and the constraint) is possibly a complicated contour and difficult to be determined. Therefore, in order to simplify our problem we use real part of \( Z_{h\_ave}(z) \) \( (\text{Re}(Z_{h\_ave}(z))) \) and imaginary part of \( Z_{h\_ave}(z) \) \( (\text{Im}(Z_{h\_ave}(z))) \) instead of \( b_{ave} \) and \( k_{ave} \).

\[
J = \frac{1}{2} \left[ \frac{\text{Re}(Z_{h\_ave}(z)) - \text{Re}(Z_{h\_ave\_max}(z))}{\text{Re}(Z_{h\_ave}(z)) - \text{Re}(Z_{h\_ave\_max}(z))} \right]^2
\]

\[
+ \left[ \frac{\text{Im}(Z_{h\_ave}(z)) - \text{Im}(Z_{h\_ave\_max}(z))}{\text{Im}(Z_{h\_ave}(z)) - \text{Im}(Z_{h\_ave\_max}(z))} \right]^2 \tag{13}
\]

where the constraint is the same as constraint equation (12). The terms \( \text{Re}(Z_{h\_ave\_max}(z)) \) and \( \text{Im}(Z_{h\_ave\_max}(z)) \) are the maximum real part and imaginary part of the searching space, respectively. Meanwhile, two conditions, i.e.: \( \text{Re}(Z_{h\_ave\_max}(z)) \geq \text{Re}(Z_{h\_ave\_max}(z)) \)

and \( \text{Im}(Z_{h\_ave\_max}(z)) \geq \text{Im}(Z_{h\_ave\_max}(z)) \) must be satisfied in order to ensure that the minimum point is located in the searching area. Instead of finding the minimum difference between \( Z_{h\_ave}(z) \) and

\[
Z_{h\_ave}(z) \], the new objective is to find the minimum difference between \( \text{Re}(Z_{h\_ave\_max}(z)) \) and \( \text{Im}(Z_{h\_ave\_max}(z)) \), and between \( \text{Im}(Z_{h\_ave\_max}(z)) \) and \( \text{Im}(Z_{h\_ave\_max}(z)) \).

This objective function, plotted in figure 5, is a quadratic function which has only one minimum point at \( [\text{Re}(Z_{h\_ave\_max}(z)), \text{Im}(Z_{h\_ave\_max}(z))]=[\text{Re}(Z_{h\_ave\_max}(z)), \text{Im}(Z_{h\_ave\_max}(z))] \). From equation (25), the constraint is an inequality linear constraint, thus the feasible region can be directly drawn on the contour of the objective function. Such that, there are only two possible cases which are \( \text{Re}(Z_{h\_ave\_max}(z)) \geq 0 \) and \( \text{Re}(Z_{h\_ave\_max}(z)) < 0 \).

Case 1: \( \text{Re}(Z_{h\_ave\_max}(z)) \geq 0 \)

The minimum point which is the solution is \( [\text{Re}(Z_{h\_ave\_max}(z)), \text{Im}(Z_{h\_ave\_max}(z))]=[\text{Re}(Z_{h\_ave\_max}(z)), \text{Im}(Z_{h\_ave\_max}(z))] \)

Case 2: \( \text{Re}(Z_{h\_ave\_max}(z)) < 0 \)

The minimum point which is the solution is \( [\text{Re}(Z_{h\_ave\_max}(z)), \text{Im}(Z_{h\_ave\_max}(z))]=[0, \text{Im}(Z_{h\_ave\_max}(z))] \)

After \( \text{Re}(Z_{h\_ave\_max}(z)) \) and \( \text{Im}(Z_{h\_ave\_max}(z)) \) are determined, \( b_{ave} \) and \( k_{ave} \) which are the actual variables that we want to determine can be easily derived by follow these steps. Step 1:

\[
Z_{h\_ave}(z) = \text{Re}(Z_{h\_ave\_max}(z)) + j\text{Im}(Z_{h\_ave\_max}(z)) \tag{14}
\]

Step 2:

\[
Z_{e\_ave}(z) = \frac{Z_{h\_ave}(z) - Z_d(z)}{ZOH(z)} \tag{15}
\]

Step 3:

\[
Z_{e\_ave}(z) = \frac{Z_e(z)Z_{e\_ave}(z)}{Z_e(z) - Z_{e\_ave}(z)} \tag{16}
\]

Step 4:

\[
b_{ave} = \left[ \frac{\text{Re}(Z_{ave}(z)) - \frac{k_{ave}\text{Re}(s)}{(\text{Re}(s)^2 + \text{Im}(s)^2)}}{\frac{T(z-1)}{2(z+1)}} \right] \tag{17}
\]

3.2 Implementation of adaptive virtual coupling

In order to implement the adaptive virtual coupling, there are two components, i.e. the interacting frequency estimator and the adaptation procedure, added in the system as illustrated in figure 4. The parameters of the
virtual coupling, i.e. \( b_{\text{avc}} \) and \( k_{\text{avc}} \), are frequency dependent. Therefore, the frequency, which represents the response of human perceived impedance of haptic system with adaptive virtual coupling (\( Z_{h \_\text{avc}}(z) \)), has to be determined in order to be used for adjusting the parameters of the virtual coupling. This frequency, which is called interacting frequency (IF), could be a point or a range of frequency characterized by haptic interaction.

From the parameters of the realistic performance boundaries in [10] the parameters of the adaptive virtual coupling with respect to frequency would be determined. From Figure 5, short-dash-line is a plot of human perceived impedance of a haptic interface with a static virtual coupling. The long dash-line is a plot of human perceived impedance of a haptic interface without virtual coupling (\( Z_h(z) \)) while the solid-lines are upper and lower boundaries. The dash-dot-line is a plot of human perceived impedance of a haptic interface with adaptive virtual coupling (\( Z_{h \_\text{avc}}(z) \)), while the dotted-line is the desired impedance.

From these results, two criteria that we concerned are the stability and the realistic performance of the haptic system. The objectives of an adaptive virtual coupling are to eliminate or reduce the unrealistic of haptic interaction caused by the static virtual coupling while ensuring system stability. From figure 5e plot of Re(\( Z_{h \_\text{avc}}(z) \)) (dash-dot-line) shows that this term is greater than zero, therefore the system satisfies the stability criteria. From figure 5b the plots of human perceived impedance of a haptic interface with adaptive virtual coupling (\( Z_{h \_\text{avc}}(z) \)) are located in the realistic performance boundaries. Thus these systems are said to be realistic haptic systems based on the concept of the realistic performance boundaries. Figure 5e show that in case of no virtual coupling, the real-part of the realistic performance boundaries are negative thus Re(\( Z_{h \_\text{ds}}(z) \)) < 0. As these consequences, from equation (12) the Re(\( Z_{h \_\text{avc}}(z) \)) > 0 must be set to be greater than zero in order to ensure system stability.

**Figure 4. Schematic Diagram of a Haptic System with an Adaptive Virtual Coupling**

**Figure 5 Plot of the Results on the Determination of the Adaptive Virtual Coupling of virtual environment \( (b_e = 10 \, \text{N.s/m}, \, k_e = 100000 \, \text{N/m}) \) (a) Plot of Human Perceived Impedance, (b) Enlarged Figure of Figure 5a, (c) Plot of \( b_{\text{avc}} \), (d) Plot \( k_{\text{avc}} \) and (e) Plot of Re(\( Z_h(z) \))

**4. Experiment : Evaluating the Realistic Performance of Adaptive Virtual Couplings**

The experiment was performed to qualitatively evaluate the performance of the adaptive virtual coupling system in comparison to the static virtual coupling. Our virtual environment was a virtual wall as shown by a dotted line in figure 6. The High Bandwidth Force Display Device from University of Washington [9] was used as the haptic interface device. Based on the realism of interaction of each virtual environment, users were asked to match one of the simulations (VEC1 or VEC2) with the reference virtual environment (Ref VE). The VEC1 and VEC2 were randomly assigned to be the virtual environment with static virtual coupling or with adaptive virtual coupling. Such that, the user had no bias in the experiment. The virtual environment VE1-7 has various mechanical properties ranging from low to high stiffness respectively. The stability of the interactions was observed based on the convergence of the interaction velocity. The system is said to be a stable system if the velocity, eventually, converges to zero. The results are presented in the Table 1.
5. Conclusions

In this research, a method of adapting the parameters of the virtual coupling according to the virtual environment which can enhance the realistic performance of haptic systems was proposed. A desired impedance based on realistic performance boundary is proposed in order to be used as our objective impedance. The parameters of the virtual coupling can be determined with respect to frequency. Therefore, each defined virtual environment has its own set of parameters of virtual coupling that both ensure system stability and increase the realistic performance of system. The experimental results showed that the realistic performance of haptic system can be increased by using the adaptive virtual coupling especially with the high stiffness virtual environment.

References